**How to understand matrices multiplication**

* **matrices** can be considered as a set of several **vectors** and a set of **multipliers**
* **product** can be considered as a set of several **combinations** of one matirx
  + **column vectors**:
    - \*
    - vectors: carried by the **first matirx**.
      * vectors have **the same dimensions**
      * its **dimensions don’t change** after it was calculated
    - multipliers: carried by the **second matrix**
      * one multiplier combines all vectors
        + 1) multiply all components by their corresponding vectors
        + 2) add up the results
        + produces a combination with same dimension as the vector
      * the dimension of multiplier = the dimension of vector
        + therefore: the number of columns of the **first matrix** must equal the number of rows of the **second matrix**
      * the **number** of multipliers decides the **size** of product
  + **row vectors**:

\*

* + - vectors: carried by the **second matirx**
    - multipliers: carried by the **first matrix**
    - number of vectors of product = the number of **rows of first matrix**
  + **three different ways** to understand matrix multiplication
    - Each **entry** of ***AB***:
      * the product of a **row** from ***A*** and a **column** from ***B***
      * (***AB***)ij = (row i of ***A***) times (column j of ***B***)
    - Each **column** of ***AB***
      * a combination of the columns of ***A***
      * a product of matrix(***A***) and a column from ***B***.
        + column j of ***AB*** = ***A*** times (column j of ***B***)
    - Each **row** of ***AB***
      * a combination of the rows of ***B***
      * a product of a row from ***A*** and matrix(***B***)
        + row i of ***AB*** = (row i of ***A***) times ***B***
  + **associative law**: ***ABC*** = ***A***(***BC***)
  + **commutative law**: ***FE*** =! ***EF***
  + **distributive law**: ***A***(***B***+***C***) = ***AB*** + ***AC***; (***B***+***C***)***D*** = ***BD***+***CD***

**Matrix Notation**

* m by n matrix(**rectangular**)
  + m = n [**square matrix**]: the number of m(rows) = the number of n(columns)
  + **addition *A* + *B*:** ***A*** and ***B*** must have the same shape.
* **matrices multiplication**
  + **row times column**---> **inner product**(scalar)
    - *1 by n* matrix(row vector) **x** *n by 1* matrix(column vector)
  + **notation: a**ij
    - first subscript: **row** number (1-m)
    - second subscript: **column** number (1-n)
    - **sigma notation**: **one entry**
      * ***[AB]ij = Ai1B1j + Ai2B2j + … + AinBnj = AinBnj***
* **Identity matrix(square matrix)**
  + **1s** on the right-top diagonal and 0s everywhere else
* **Permutation matrix**(exchanging the rows or columns of **I**)
  + the same rows as the **identity** (in some order)
  + ***P****-1*= ***P****T*
  + **the number** of permutations of size n **==> n!**
    - row 1 has **n** choices, row 2 has **n-1** choices, last row has **1** choice
  + **row exchange matrix：*PA*** 
    - row vectors are on the right side, so multipliers are on the left side
  + **column exchange matrix：*AP***
    - column vectors are on the left side, so multipliers are on the right side
* **Inverse(a operation or a new matrix)**
  + **pronounce: *A* inverse(square matrix)**
  + **fundamental property:** *b* = **A**x***A−1****b* = ***A−1A****x****A−1****b* = *x*
    - multiply by A and multiply by A−1 , back where started
  + **definition:** ***A−1A*** =***I*** & ***AA−1*** = ***I*** 
    - The inverse of ***A*** is a matrix ***B*** such that ***BA*** = ***I*** and ***AB*** = ***I***
  + **condition**(when the inverse exists)
    - ***A*** is invertible ***A*** has n pivots
    - ***A*** matirx is invertible only if elimination produces n pivots
    - ***A*** matirx is invertible if its **determinant** is NOT zero
      * ***A*** 2 by 2 matrix is invertible if and **only if ad-bc is NOT zero**.
      * In *MATLAB*, the invertibility test is **to find n nonzero pivots**
    - **A diagonal matrix** has an inverse provided **no diagonal entries are zero**
  + **porperties**
    - 1) ***A*** matrix only have one ***A−1***
    - 2) If ***A*** is invertible, the one and only solution to ***A****x* = *b* is *x* = ***A−1*** *b*
    - 3) If ***A****x =* ***0*** *& x =!* ***0****,* then ***A*** is **NOT** invertible
      * If ***A*** is invertible, then ***A****x* =***0*** can only have the zero solution *x* =***0***
  + **The inverse of *AB***(***AB***)***−1***= ***B−1A−1*** ; (***ABC***)***−1*** = ***C−1B−1A−1***
    - the inverses come **in reverse order**
  + **Gauss-Jordan Method**(Find ***A-1***)
    - steps
      * apply **gaussian elimination** to the matrix augmented by ***I***
      * continues by subtracting multiples of a row from the rows above
        + this produces zeros **above** the **diagonal** as well as **below**
      * divide by pivots
    - reason: ***A*** and ***I*** change simultaneously, since ***A***=>***I***, ***I***=>***A-1***
    - relation to determinant: determinant is the **product of pivots**
  + **Transpose Matrix(a operation or a new matrix)**
    - **fundamental property:** (***AT***)***ij***= ***Aji***
      * **exchange** the **rows** and **columns** of the same matrix.
        + exchange **vecters** **multipliers**
        + exchange **row vecters** **column vecters**
    - **porperties**
      * **the matrix of *AB*: (*AB*)*T* = *BTAT***
        + the transposes come **in reverse order**
      * **the matrix of inverse *A*: (*A-1*)*T* = (*AT*)-1**
        + **Inverse of *AT* = transpose of *A-1***
  + **Symmetric Matrix(a new matrix)**
    - **fundamental property: each entry:** *a****ij***= *a****ji***
      * **if *A* is symmetric , then *AT* = *A***
      * **row vecters** = **column vecters**
    - **property**
      * **if *A* is symmetric, then *A-1* is symmetric as well**
    - **symmetric products**
      * ***RTR* is square symmetric matrix for any *R***
      * ***RTR* and *RRT* are square symmetric matrices**
        + ***RTR* is n by n**
        + ***RRT* is m by m**
        + ***RTR* and *RRT* are not offen equal**
      * **use for most scientific problems** 
        + **start: with rectangular matrix R**
        + **end up: with *RTR* and *RRT* or both**
        + **symmetric matrices appear in subject**

**each *action* has an equal and opposite *reaction***

**aij that gives the action of i onto j is matched by aji**